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Technical Note TN-7

APPLICATIONS OF TRANSIENT ELECTROMAGNETIC TECHNIQUES

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INTRODUCTION

The ever increasing requirements for greater depth of exploration, improved rejection of conductive overburden or host-rock response and enhanced definition of potential ore-bearing structures all demand the application of large scale transient electromagnetic methods with their inherent ability to generate highly diagnostic data.

Furthermore it is not generally realized that transient electromagnetic techniques lend themselves very well to general geological mapping. Thus, in addition to their employment for the direct detection of conductive ores, they can also be used in the relatively resistive environments that often occur when prospecting for aggregates or aquifers, or when measuring the depth to bedrock or the thickness of permafrost. Indeed transient electromagnetic techniques are one of the "unconventional" methods employed in Russia for exploration for oil.

This short technical note reviews certain aspects of transient techniques which are not well covered in the literature. In the first section a brief outline of the nature of the transient response is followed by a short description of a typical large transient system, after which the reasons for certain design features are summarized. The next section illustrates the response calculated for several theoretical models and discusses the difference between confined and unbounded conductors. The last section considers several popular misconceptions about transient techniques and considers some merits and deficiencies.

The author wishes to acknowledge that much of the material in Section III either originated with or was supplied by Dr. Alex Kaufman of the Colorado School of Mines. More importantly the author wishes to express his gratitude for the many hours spent discussing electromagnetic techniques with Dr. Kaufman, whose insight is invariably illuminating.

SYSTEM CONSIDERATIONS

A procedure commonly utilized for ground exploration using transient techniques is to lay a large loop (as shown in Fig. 1) in the vicinity of the area to be examined. A steady current is caused to flow in the loop for a sufficiently long time to allow turn-on transients in the ground to dissipate. The quiescent current is then sharply terminated in a controlled fashion; for example the current turn-off may be a linear-ramp waveform (Fig. 2a). In accord with Faraday's Law rapid reduction of the transmitter current, and thus also of the transmitter primary magnetic field, induces an electromotive force (emf) in nearby conductors. The magnitude of this emf is proportional to the time rate-of-change of the primary magnetic field at the conductor. For this reason it is desirable to reduce a large transmitter current to zero in a short time so as to achieve a large emf of short duration (Fig. 2b). This emf causes eddy currents to flow in the conductor with a characteristic decay which is a function of the conductivity, size, and shape of the conductor. The decaying currents generate a proportional (secondary) magnetic field (Fig. 2c), the time rate-of-change of which is measured by a receiver coil.

Analysis of the nature of the transient decay is carried out by sampling the amplitude at numerous intervals of time (and over many cycles of the transmitter pulse so as to enhance the signal-to-noise ratio). A typical system block diagram is illustrated in Fig. 3.

The survey output is an accurate knowledge of the three spatial components of the time-derivative of the secondary magnetic field as a function of position on the earth's surface and from this data much information can usually be derived about a survey target.

In order to appreciate certain system constraints, suppose for simplicity that the target is a wire loop having resistance and inductance as shown in Fig. 4. The transmitter primary magnetic field coupling with the loop generates a flux which is caused to rapidly decrease from a steady state value ϕ_p to zero within a time Δt which is much less than the time constant of the loop (defined as L/R). The emf generated in the loop is given by $\phi_p/\Delta t$ and from simple circuit theory the current can be shown to be

$$i(t) = \frac{\phi_p}{L} e^{-t/\tau} \qquad \text{where } \tau = \frac{L}{R} \tag{1}$$

The secondary magnetic field is proportional at all times to i(t).

Note that the initial amplitude of both the current and the secondary magnetic field at t = 0 is independent of the ring resistance. Since by definition the inductance of the loop is the flux per unit current, we are not surprised to see that the initial current is given by ϕ_p/L i.e. it is exactly that value required to make the total flux through the loop equal to the value that existed just before transmitter turn-off.

The receiver-coil output voltage is proportional to the time rate-of-change of the secondary magnetic field and is therefore of the form

$$e_0 \propto \frac{1}{\tau} e^{-t/\tau} \tag{2}$$

We see from this expression that conductive targets (i.e. those having a small value of resistance and thus a large value of τ) yield signals with small initial amplitude that, however, decay relatively slowly. Signals from poorly conducting targets (large resistance) have high initial amplitude and decay rapidly. Typical decay times for orebodies range from as small as 100–200 μsec to as high as 10–20 msec and thus cover a factor of approximately one hundred in

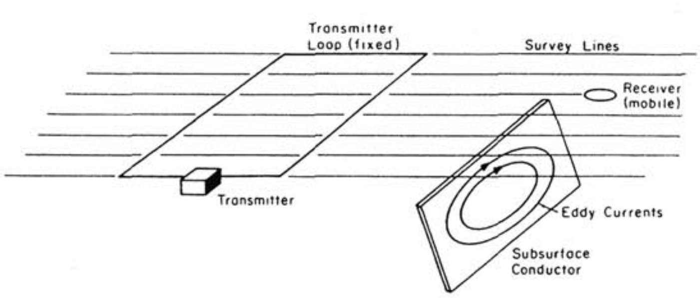


FIGURE 1. Survey configuration.

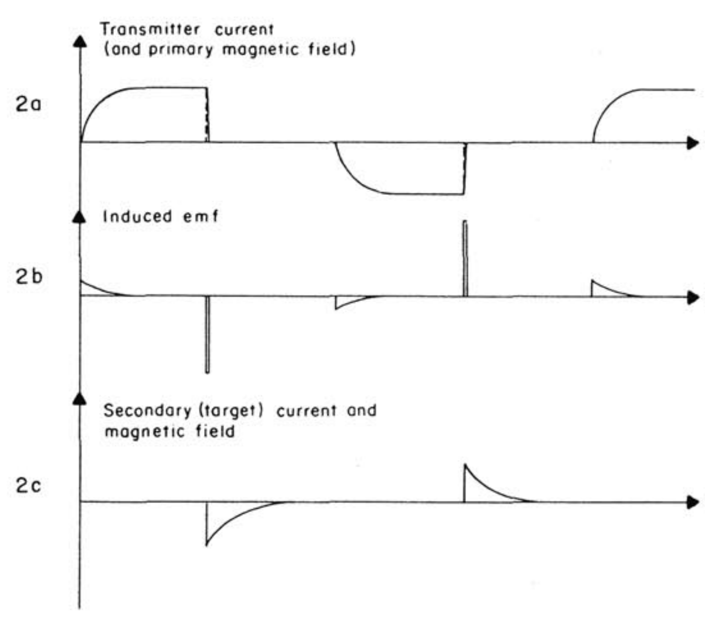


FIGURE 2. System waveforms.

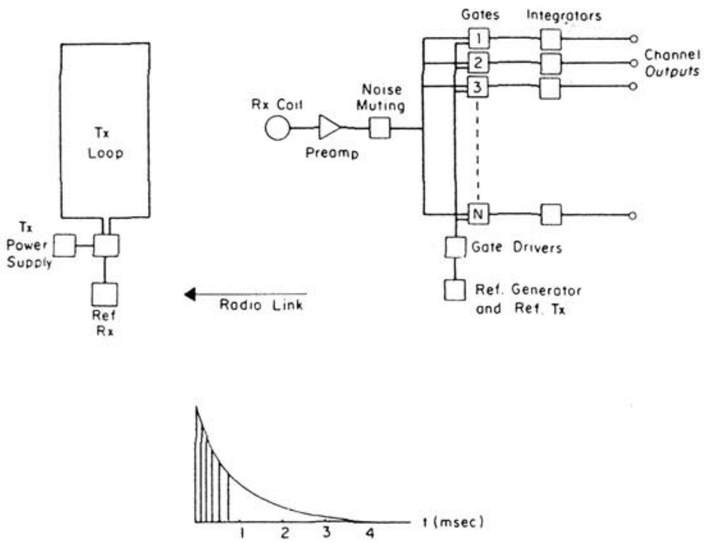


FIGURE 3. Block diagram and typical transient (showing initial gates).

time. In order to accurately resolve this range of transient decays, measurements must be made at a large number of time intervals. Furthermore at the initial portion of the transient decay the receiver output voltage varies rapidly with time, particularly for small values of τ. For this reason the sampling "gates", particularly the earlier ones, must be of short duration so as to avoid distortion of the signal resulting from appreciable changes in the signal amplitude during the sampling period. Since the transients obtained in actual practice can be quite complex the only way to ensure low distortion is to employ a large number of narrow gates.

As equation (2) indicates the initial amplitude of the transient decay can also vary by a factor of one-hundred arising simply from the variation of τ; target size, depth, etc., can easily account for another factor of one hundred, so it is most important that transient measuring systems have a very large dynamic range to handle the wide variety of signals that will occur in the field.

It was assumed above that the turn-off time of the transmitter pulse was much shorter than the characteristic decay time of the targets to be measured. In the event that this condition is not met for targets exhibiting exponential decay with low values of τ the amplitude of the response will be less than that given in equation (2). Thus the response from poor conductors will be suppressed when compared with that from good conductors. Such suppression might be an advantage in a system designed to search only for better conductors but it is a definite disadvantage in a system designed to (i) search for *all* types of conductors and (ii) additionally map geology since in this application it materially complicates the data interpretation. For these reasons it is important to maintain the transmitter turn-off time as short as possible.

Finally it should be noted that by their very nature transient systems are broadband. A Fourier analysis of the emf induced in the target (Fig. 2b) shows that odd-harmonic components exist from the basic pulse repetition-frequency (typically from 3 to 30 Hz) up to a *value determined by the duration of the transmitter turn-off time; for* example a 200 μsec turn-off puts significant excitation to frequencies of many kHz. This multi-spectral excitation is very desirable for obtaining detailed information about the target spectral response, however an accompanying disadvantage is that the receiver, being broadband, is susceptible to external noise and interference. In fixed-frequency systems, where the primary field is carefully cancelled out, the system noise is usually determined by errors in this cancellation and a point is soon reached where increases in transmitter dipole moment do not increase the signal-to-noise ratio. The situation is quite different for transient systems in which measurement is made during the transmitter off-time; for such systems the effective exploration depth is often set by external noise and the availability of a large transmitter loop with a high value of transmitter current becomes of great importance. Other techniques such as synchronization of the system to power-line frequencies so as to avoid having the system output oscillating at the powerline frequency and "spherics" noise-limiters also help reduce the sensitivity to external noise and interference and increase the exploration depth.

TARGET RESPONSES

I Confined Targets

As our first example of a target response consider the case of a conducting sphere which, for the sake of simplicity, we will assume to be in a region of uniform magnetic field which is suddenly terminated at time $t = 0$. The sphere, of radius a and conductivity σ, is shown in Fig. 5.

At the instant after termination of the primary field (called the early-time) currents immediately flow on the surface of the sphere with a distribution that exactly maintains the original uniform magnetic field within the sphere. The current distribution at this time (indicated in Fig. 6a) is independent of the conductivity of the sphere and we say that we are in the high-frequency limit since the current distribution is that which would flow if the sphere were located in a very high-frequency alternating (uniform) magnetic field. The definition of high-frequency in this case is such that the electrical skin-depth in the sphere material

$$\delta = \left(\frac{2}{\mu\sigma\omega}\right)^{1/2} \tag{3}$$

$$\text{where } \mu = 4\pi \times 10^{-7}\ \text{h/m}$$
$$\omega = 2\pi f$$
$$f = \text{frequency (Hz)}$$

is much less than the sphere radius.

At time $t = 0^+$ the primary field has disappeared and for all further time the distribution of circulating currents within the sphere is governed solely by their interaction with their own magnetic field.

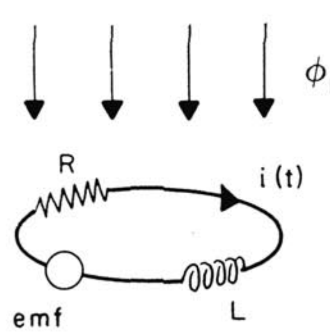


FIGURE 4. Wire loop target.

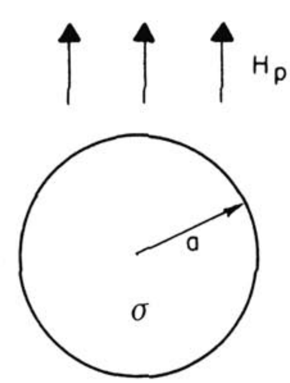


FIGURE 5. Sphere target.

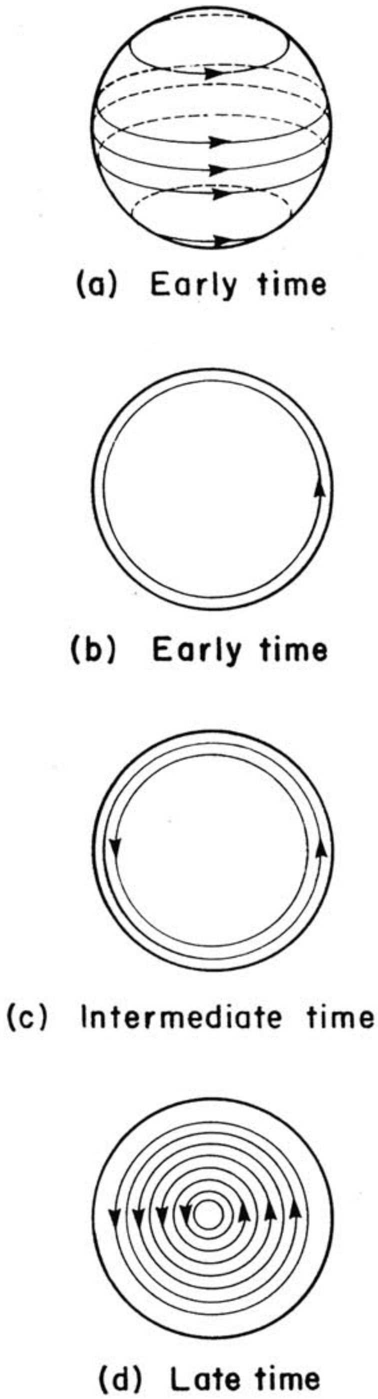


FIGURE 6. Sphere currents at various times (a – surface currents; b–d – equatorial/plane currents).

Consider then Fig. 6b which shows in section the current flow in the equatorial plane of the sphere at $t = 0^+$. The current, as mentioned above, is concentrated at the edge of the sphere. Virtually immediately, however, the amplitude of the current commences to decrease as a result of ohmic losses in the conductive sphere material. The local magnetic field from the current also decreases and (again through Faraday's Law) this decrease induces an emf which causes new current to flow, as shown in Fig. 6c. This process continues, with the result that the current flow moves inward with the passage of time. We say that the current diffuses radially inwards, although at no time is there actually a radial component of current flow – we are simply seeing the interaction of currents and their magnetic field in a conductive body.

The period of time during which the actual current distribution is in motion is called the "intermediate-time". During this period the external magnetic field associated with the moving and decreasing currents decays rapidly with time. A stage is reached, however, where the current distribution becomes invariant with time, with the form indicated in Fig. 6d. Close to the sphere centre the current density increases linearly with radial distance, becoming relatively uniformly distributed at one-half the radius, and decreasing slightly towards the edge. This period of time is called the "late-time"; the inductance and resistance associated with each current ring have stabilized and from this time onwards both the currents and their associated external magnetic field commence to decay exponentially with a time-constant given by

$$\tau = \frac{\sigma \mu a^2}{\pi^2} \tag{4}$$

A plot of the distribution of current with time is shown in Fig. 7 and the time derivative of the external magnetic field is shown in Fig. 8.

We see that measurement of the transient decay or its derivative yields useful diagnostic information about the electrical properties of the target. The late stage of the decay commences at a value of $t/\tau \simeq 0.5$ (characteristic of a sphere) and determination of τ gives a value for σa^2.

Furthermore, measurement of the transient response at various locations in the vicinity of the target (made simpler by the absence of the primary field) additionally yields information as to the sphere

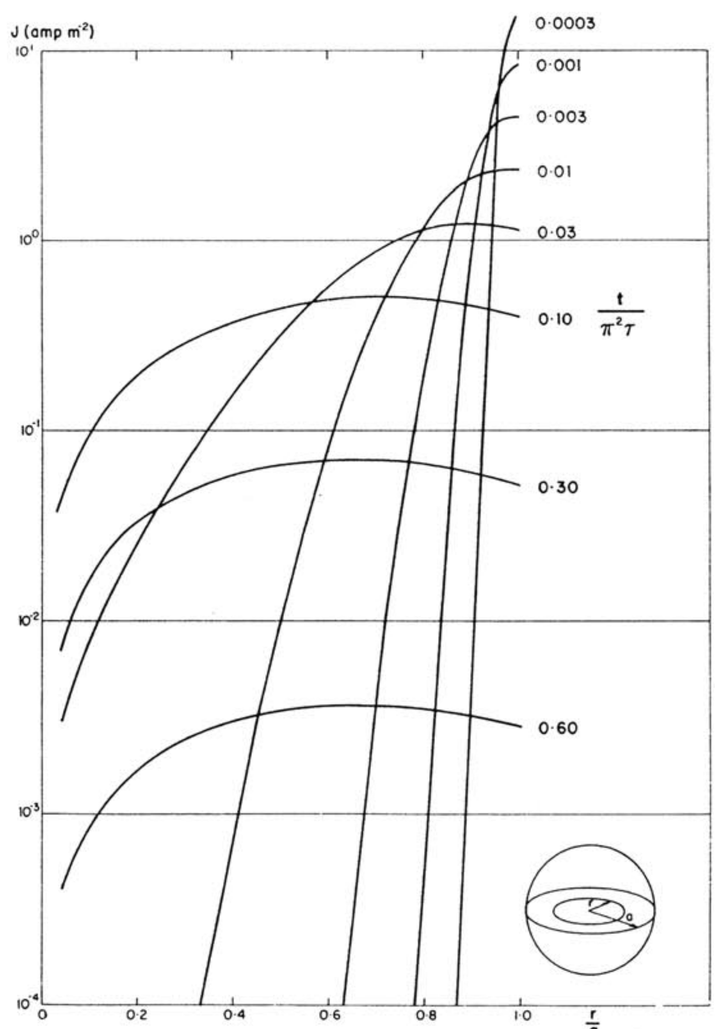


FIGURE 7. Radial distribution of sphere currents.

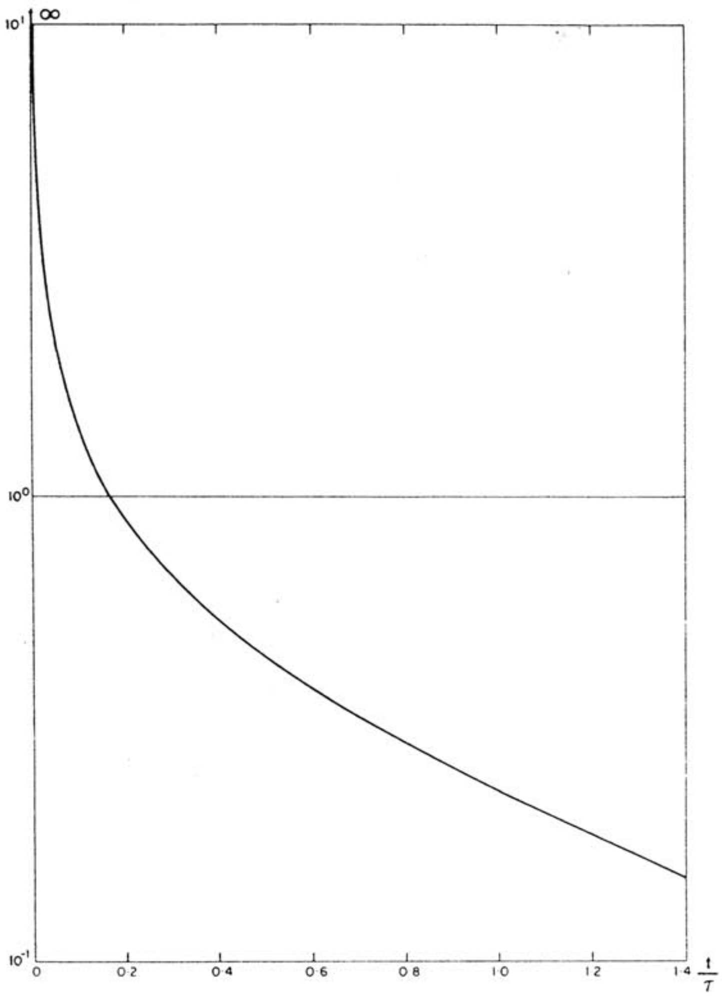


FIGURE 8. Time derivative of sphere secondary magnetic field.

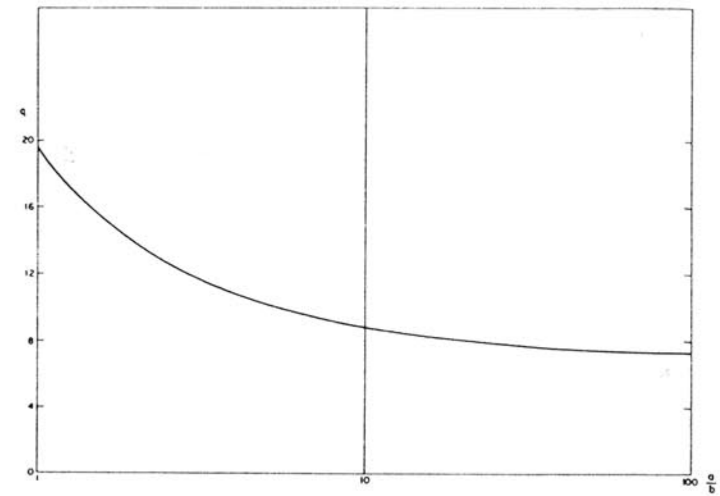


FIGURE 10. Dependence of spheroid time-constant on eccentricity.

radius and depth. This is particularly true of the early channels since, as we have seen above, the early current flow is independent of the sphere conductivity and dependent only on its geometry. Our sphere is an example of an isolated conductor, i.e. a confined target surrounded by an insulator. The significance of this point is that for any confined conductor at early times the current distribution is a function of time and the overall decay is not exponential; there is however always some stage of time after which the current distribution becomes invariant with time and the decay becomes exponential at a rate determined by the shape, size, and conductivity of the body.

A second model that is useful is the oblate spheroid shown in Fig 9. Again we assume that the energizing field is uniform (a condition that, as we shall see later, is usually not too restrictive). As for the case of the sphere immediately after removal of the primary field surface currents, concentrated on the surface, flow so as to maintain the internal magnetic field at the value of the original (uniform)

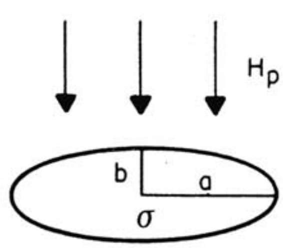


FIGURE 9. Oblate spheroid.

inducing field. These currents diffuse radially inwards for a period of time which depends essentially on σ and a, after which the current distribution is no longer a function of time and the currents and external magnetic field decay exponentially with a characteristic time-constant.

More specifically the late-time commences at approximately (1)

$$\frac{d}{a} \simeq 1.5 \tag{5}$$

$$\text{where } d = 2\pi\left(\frac{2t}{\mu\sigma}\right)^{1/2} \tag{6}$$

The quantity d is a characteristic diffusion distance which gives the position of the current at time t and about which more will be said later. This quantity is the transient analogue of the skin-depth defined in equation (3).

Combining equations (5) and (6) we see that the late-time is given by

$$t_l = \frac{1.5^2\mu a^2\sigma}{8\pi^2} = 3.6 \times 10^{-8} a^2\sigma \tag{7}$$

and is a function of σ and a^2 but not b.

The transient decay time-constant is given by (1)

$$\tau = \frac{\mu S a}{q} \tag{8}$$

where $S = 2\sigma b$ (the conductivity-thickness product)

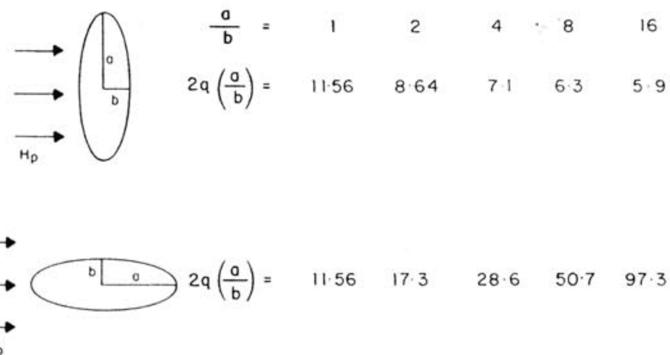
and the function q(a/b) is plotted in Fig. 10. From this figure we see that for the larger values of eccentricity a/b the value of q is approximately 8 and the time-constant is therefore determined essentially by S and a as shown by equation (8). Furthermore subject to the assumption of the constancy of q, equations (7) and (8) can be combined to yield

$$\frac{t_l}{\tau} = 0.11\frac{a}{b}, \quad \frac{a}{b} > 4 \tag{9}$$

to give the aspect-ratio of the body. This equation shows that a highly eccentric spheroid exhibits a late time which is comparable to or larger than its time-constant.

An eccentric oblate spheroid resembles massive sulphide orebodies, making this model useful in exploration for such deposits. As an example assume that the conductivity of the ore material is 2 mhos per meter, the radius a is 150 m and the thickness 2b is 20 m. Then the conductivity-thickness product s is 40 mhos, late-time begins at t_l = 1.5 msec and the decay time-constant is 0.94 msec. Such values are quite typical.

It was mentioned above that the assumption of a uniform primary field was not usually too restrictive. Consider again the case of a



$$\tau = \frac{\mu S a}{2q\left(\frac{a}{b}\right)} \quad , \text{ where } S = 2\sigma b$$

FIGURE 11. Time constant – infinite elliptical cylinder.

sphere excited by a uniform magnetic field. As is well known the resulting external magnetic field distribution arising from the sphere currents at any instant of time is that of a magnetic dipole located at the centre of the sphere. It is this apparent dipole moment which decays, initially faster than exponentially, then exponentially.

If now the sphere is excited by means of an adjacent magnetic dipole rather than a uniform magnetic field higher order multipoles are also induced. The strength of each multipole is a function of the distance (in terms of the sphere radius) of the inducing dipole from the sphere. Fortunately (i) the higher order multipole fields decrease more rapidly than the dipole field with measurement distance from the sphere and (ii) the higher order multipoles also decay more rapidly in time than the dipole (2). For these reasons, as long as both the inducing dipole and the receiver coil are further than approximately two sphere radii from the sphere centre those currents in the sphere (or indeed, any compact target) causing the secondary field sensed by the receiver coil will be essentially the same as if the inducing field were uniform by the time that the late-time condition has arrived; a very useful result!

Another phenomenom that will be observed occurs if an eccentric target such as the spheroid is subject to a primary field at an oblique angle. The inducing field can be resolved into components along each axis a and b. The late-stage time-constant for currents circulating about the b axis is however greater than the time-constant for currents circulating about the a axis. The result is a decaying dipole whose axis changes direction with time, eventually pointing in the direction that corresponds to the largest time-constant and thus largest projected area, independently of the orientation of the inducing dipole. Conversely a target exhibiting spherical symmetry generates a dipole moment that is always aligned with the primary field direction at the target.

This effect can be deduced from Fig. 11 which gives the time-constants for an infinite elliptical cylinder oriented so as to be both parallel and perpendicular to a uniform primary magnetic field (3). It is seen that for reasonably eccentric cylinders there is an order of magnitude difference in the late-time decay rate. Although this example assumes a uniform inducing field it can also be shown (4) that the time-constants listed in the figure are approximately correct for magnetic dipole excitation as long as both the source and observation point are located a distance of several times a from the target.

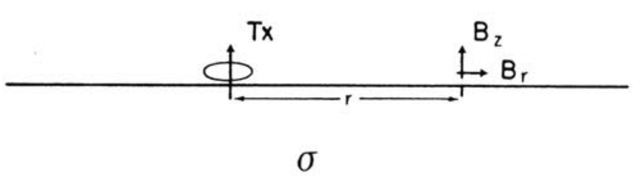


FIGURE 12. Homogeneous half-space.

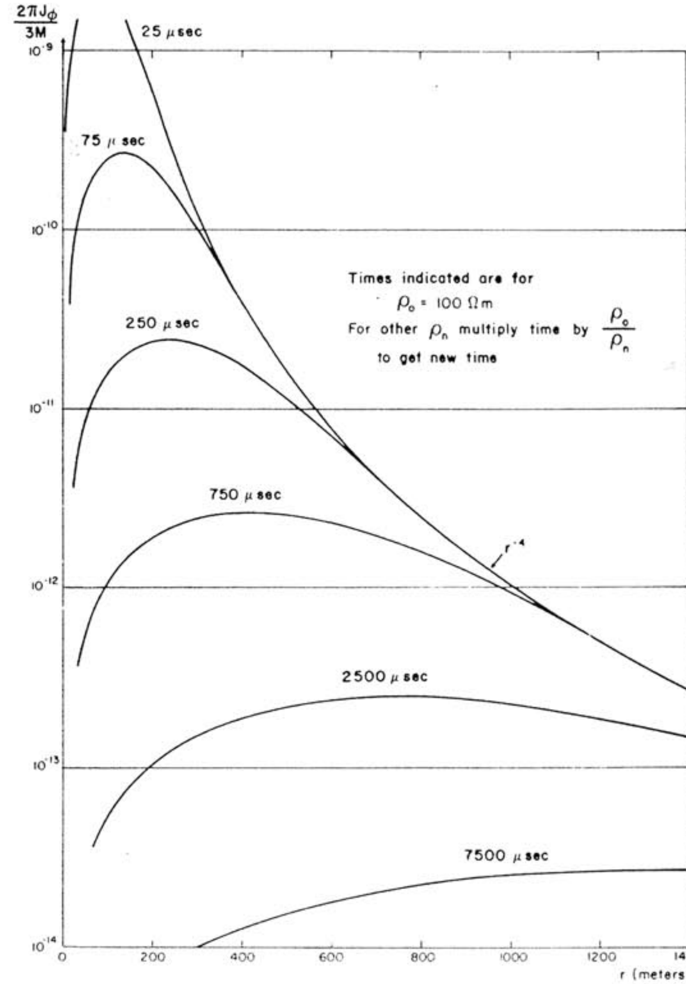


FIGURE 13. Current density – surface homogeneous half-space.

II Homogeneous Half-space

Consider a vertical magnetic dipole transmitter located on a homogeneous half-space as shown in Fig. 12. At time $t = 0$ the inducing dipole moment is reduced instantly to zero. Immediately* a surface current flows, distributed in such a manner as to maintain the magnetic field everywhere at the value that existed before turn-off: such a surface current flow decreases as r^{-4} with distance from the transmitter.

Near the transmitter the surface current starts to diffuse into the homogeneous half-space, whereas the current at great distance maintains the value dictated by the r^{-4} fall-off as illustrated in Fig. 13. The net effect as time progresses is that, due to the deficiency of current near the transmitter, the current appears to have moved out and down as a diffusing ring. The real cause of this effect is the relative decay of the current at the apparent trailing edge of the ring. Such a current distribution was recently described by Nabighian (5) whose results are shown in Fig. 14.

The magnitude of the surface current induced at time $t = 0^+$ is not a function of the conductivity. However the apparent velocity with which the ring expands away from the transmitter is inversely proportional to the conductivity as indicated on Fig. 13.

It is useful to plot the radial position of the surface current maximum as a function of time for various resistivities (Fig. 15) since a comparison of Figs. (13) and (14) shows that the position of this maximum also accurately locates the radial distance of the subsur-

*It should be noted that only diffusion effects are of any significance at the intervals of time that are commonly used for geophysical measurements (the quasi-stationary approximation). Wave propagation effects which occur much earlier have long since disappeared.

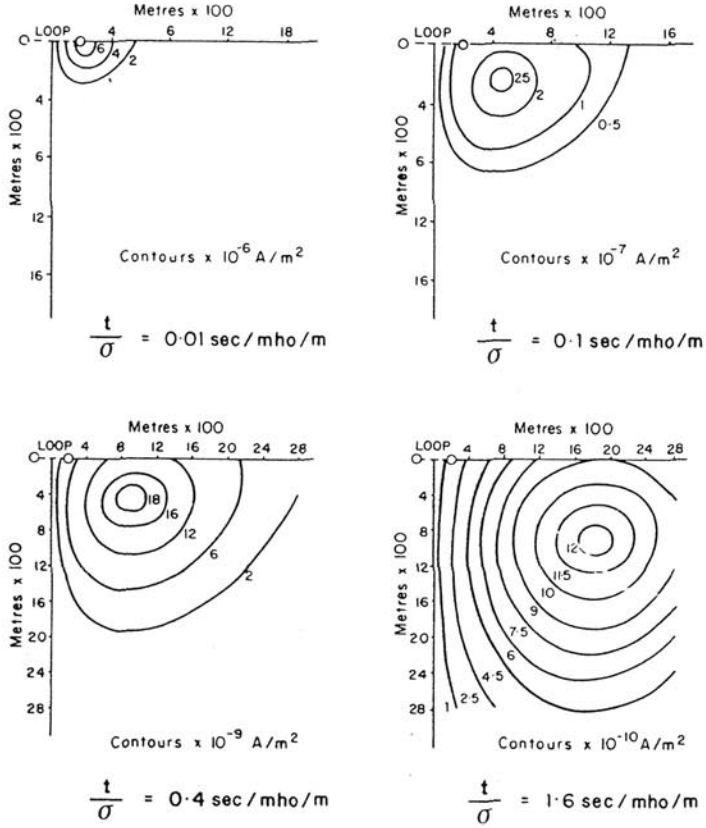


FIGURE 14. Computed contours of current density passing through loop centre (loop has dimensions 400 × 800 m (5)).

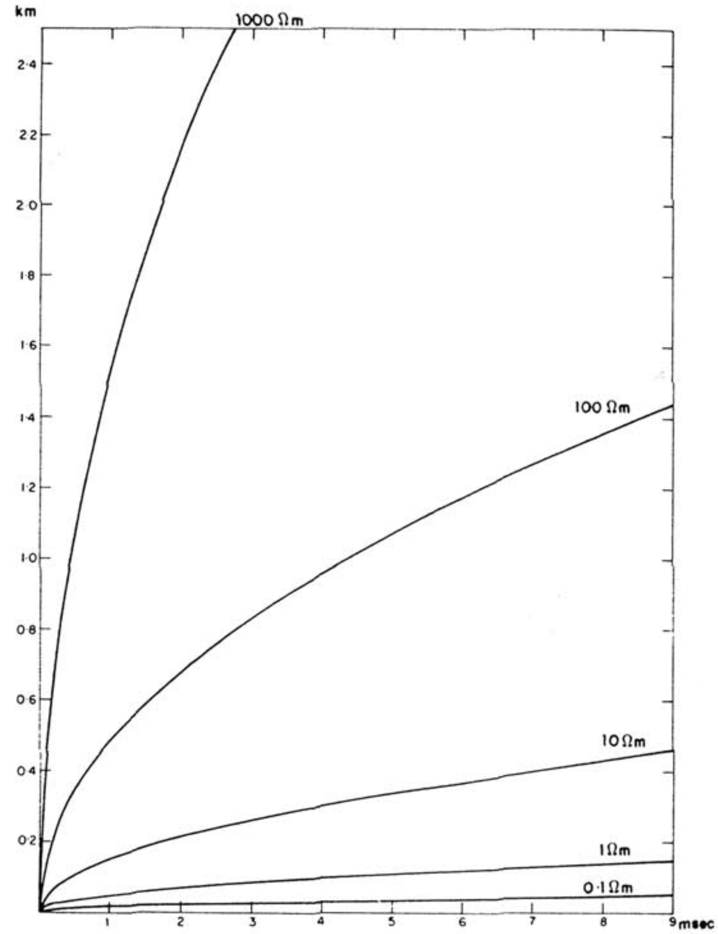


FIGURE 15. Location of surface current maximum–homogeneous half-space.

face current maximum. The location of the surface current maximum is proportional to

$$d = 2\pi\left(\frac{2t}{\mu\sigma}\right)^{1/2} \tag{6}$$

the characteristic diffusion length discussed in the previous section. The peak can be shown to be located at

$$r_{max} \simeq \frac{d}{5.2} \tag{10}$$

The significance of the diffusion distance d arises from the fact that it enables us to visualize the location of the maximum current density at any instant of time.

Since the location of the surface current maximum increases as $t^{1/2}$ the apparent velocity varies as $t^{-1/2}$. Initially the wave expands rapidly but as time passes the velocity decreases to a small value. The ring diameter becomes practically self-limiting. This feature is of practical importance because at the resistivities encountered in most ore materials (less than 1 Ωm) Fig. 15 shows that the current maximum travels at most a few tens of meters in any reasonable length of time. In conductive materials diffusion is a slow process; it is this fact that accounts for the relatively long time required for currents to stabilize in a confined conductor as demonstrated for the spheroid example above.

The magnetic field arising from a ring current is schematically illustrated in Fig. 16. The actual calculated magnetic field components arising on the surface (for the current distribution at 75 μsec in Fig. 13) are shown in Fig. 17 along with the current distribution. At large distance, much greater than the radius of the effective current loop, the vertical magnetic field still decreases as r^{-3} since the surface current at the large distance maintains the magnetic field at the value before transmitter current shut-off. At distances close to the transmitter the newly developed absence of current (recall that the original current density fell off as r^{-4}, so was very large near the transmitter) causes the vertical component of the magnetic field to be negative since all the current contributing to the field component now resides at relatively large radial distance. In a small region near the transmitter the vertical magnetic field is not a function of position in agreement with the well-known result for the invariance of the vertical magnetic field near the centre of a loop, illustrated in Fig. 16.

As time increases the current ring diffuses outwards and downwards; the vertical magnetic field for various times is shown in Fig. 18. When the current has moved out an appreciable distance from the transmitter location, there is a larger region where the vertical magnetic field is not a function of position, as indicated on the figure. This time is defined as the late-stage and is, of course, a function of the transmitter-receiver spacing since the current must have moved

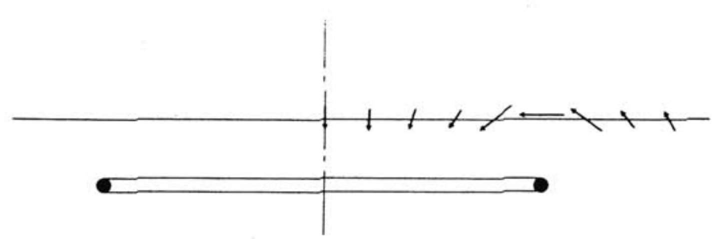


FIGURE 16. Magnetic field above current ring.

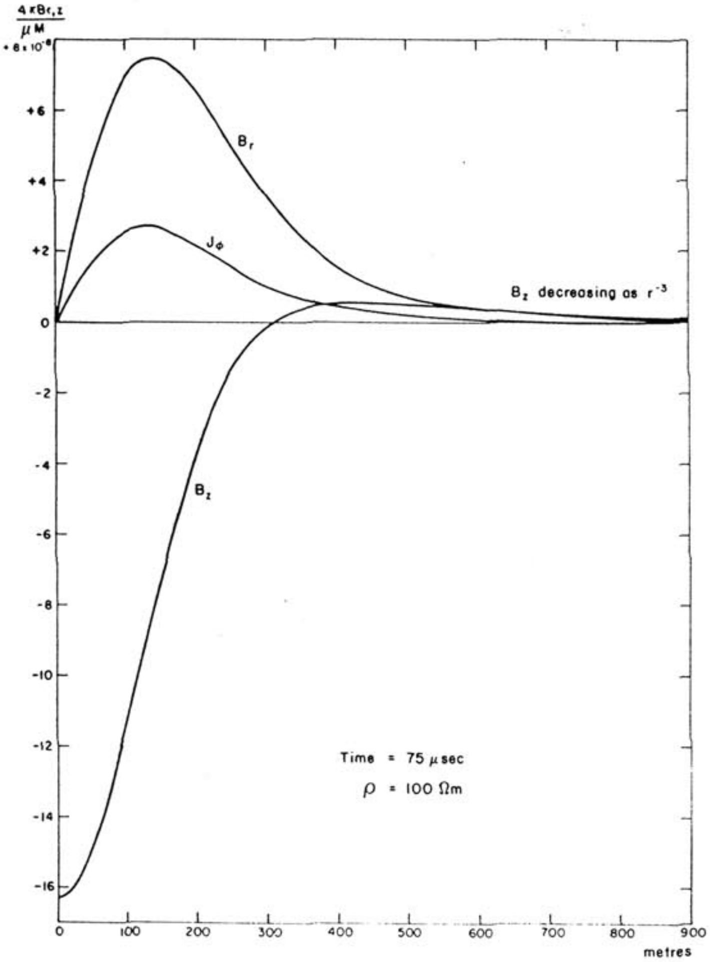


FIGURE 17. Magnetic field components at surface of homogeneous half-space.

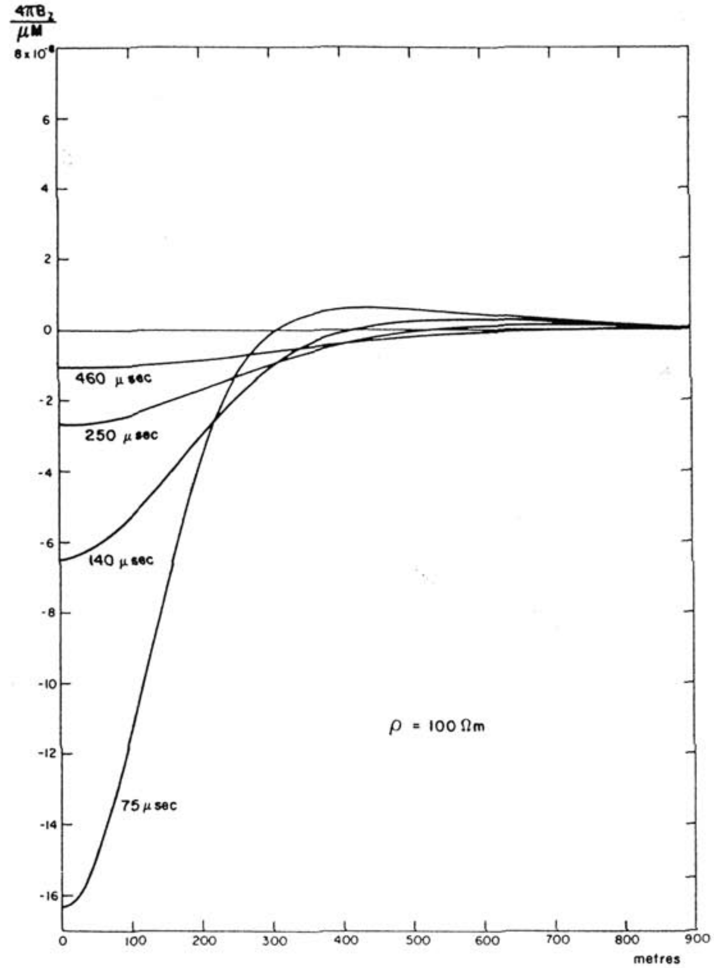


FIGURE 18. Vertical magnetic field component at surface of homogeneous half-space.

a large distance compared to this spacing. Late-time is usually considered to have commenced when

$$\frac{d}{r} > 10 \tag{11}$$

where r = Tx/Rx spacing
and d is defined in equation (6)

At such late-times a considerable simplification occurs in the expressions for the magnetic field components; they become (6)

$$B_r \simeq \frac{\mu M}{4\pi r^3}\frac{r^4}{32t^2}(\mu\sigma)^2 \tag{12}$$

$$\text{and } B_z \simeq \frac{\mu M}{4\pi r^3}\frac{2r^3}{15\pi^{1/2}}\frac{(\mu\sigma)^{3/2}}{t^{3/2}} \tag{13}$$

where M = transmitter dipole moment

These equations are the basis of what the Russians call "transient sounding in the near-zone" for determining the electrical conductivity (resistivity) of the ground. It is evident that measurement of either B_r or B_z as a function of time will permit the determination of σ. Several features suggest measurement of the vertical field component; (i) at late-times both this component and its time derivative are larger with the result that the measurement is relatively insensitive to receiver coil misorientation. (ii) The magnitude of this component is not a function of the distance r between the measurement station and the transmitter dipole. The reason for this has been discussed above and is clear from Figs. (16) and (18). (iii) This component and its derivative decrease more slowly with time in the late stage. (iv) Although this component is proportional to $\sigma^{3/2}$ whereas the radial component is proportional to σ^2, the vertical component still offers adequate resolution of small changes in terrain conductivity.

Since we are making our measurements with a coil it is the time derivatives which are of importance; they are given by

$$\frac{\partial B_r}{\partial t} \simeq \frac{-\mu M r}{64\pi}\frac{(\mu\sigma)^2}{t^3} \tag{14}$$

$$\text{and } \frac{\partial B_z}{\partial t} \simeq \frac{\mu M}{20\pi^{3/2}}\frac{(\mu\sigma)^{3/2}}{t^{5/2}} \tag{15}$$

The voltage induced in a horizontal receiver coil decays as $t^{-5/2}$, characteristic of a uniform half-space. In distinction with the earlier case of confined conductors no insulating barrier now exists to contain the currents; they diffuse without limit. The response is no longer exponential.

It should be kept in mind that the functional dependency of either field component on the terrain conductivity arises only because the apparent velocity of the diffusing current is related to the conductivity. The actual magnitude of the initial current is, as stated earlier, dependent only on the transmitter dipole moment and is independent of the ground conductivity.

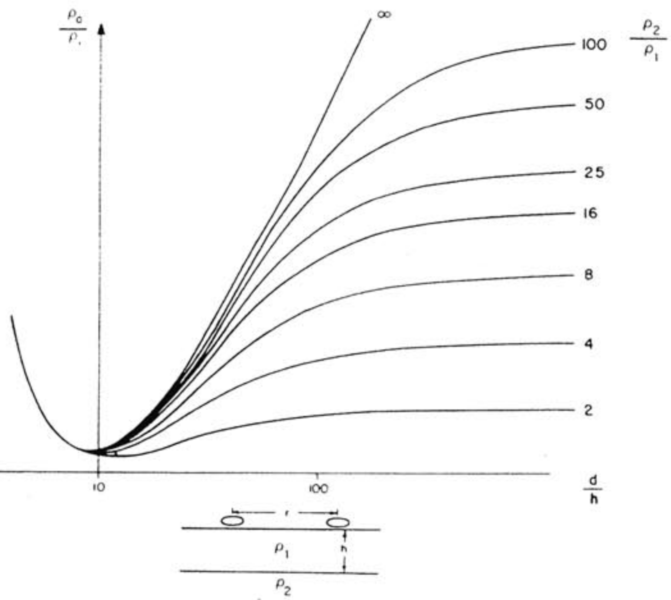


FIGURE 19(a). Two layer curves.

$\frac{\rho_2}{\rho_1} > 1, \ \frac{r}{h} = 1$

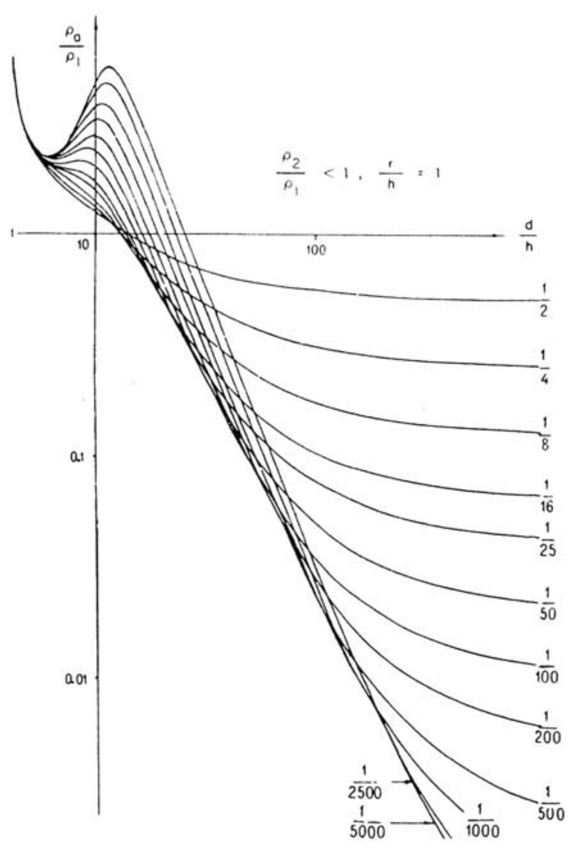


FIGURE 19(b). Two layer curves

Equation (15) suggests the possibility of inversion so that the apparent resistivity as a function of time is defined by

$$\rho_a(t) \triangleq \frac{\mu}{4\pi t}\left(\frac{2\mu M}{5t\dot{B}_z}\right)^{2/3} \tag{16}$$

If now measurements are made of $\dot{B}_z$ for late values of time and the values substituted into equation (16) the apparent resistivity so defined should not be a function of time, indicating the presence of a uniform half-space.

III Layered Earth

Suppose, however, that the earth is not uniform but consists of a layer of resistivity ρ_1 and thickness h overlying a substrate of resistivity ρ_2. It is evident that at sufficiently early time the diffusing current will be situated entirely in the upper medium and measurements of the field components will be diagnostic of that medium. Conversely at much later times the currents will be predominantly in the substrate and measurements of the magnetic field components will substantially reflect its characteristics. The behaviour at intermediate time will be diagnostic of the thickness of the upper layer.

Many calculations for a layered earth have been carried out by the Russians. Typical results are shown in Figs. (19a) and (19b) where it is seen that for both a relatively insulating or conducting substrate the apparent resistivity defined in equation (16) is a well-behaved and diagnostic feature of the electrical properties. Such curves, which are also available for a three-layered earth, are used in a fashion completely analogous to the use of master curves for conventional resistivity. One simply plots the calculated values of $\rho_a(t)$ (derived from the measured values of $\dot{B}_z$ and t) against $t^{1/2}$ (the abcissa of the curves is given in terms of d/h where d is defined in equation (6) and h is the upper-layer thickness) on log-log paper to the same scale as the curves and shifts the plotted data until agreement is achieved, whereupon ρ_1, ρ_2, and h can be calculated.

Some important features of this technique are as follows:

(i) in our earlier discussion it was noted that for a uniform half-space the measured apparent resistivity was independent of the intercoil spacing r as long as care was taken to ensure that the measurements were made at the late-stage of the transient decay (which time was governed by the resistivity). Similarly, in the case of a two-layered earth the response should be independent of the intercoil spacing as long as the same condition holds for the upper-layer resistivity. That this is essentially the case is shown by a comparison of Fig. (19a) and Fig. (20) where the major difference is seen to occur at early values of d/h.

(ii) Fig. (20), which illustrates the two layered response when the intercoil spacing is only one-quarter of the upper-layer thickness, clearly indicates that it is possible, using transient techniques, to sound to a distance which is much greater (four times, in this case)

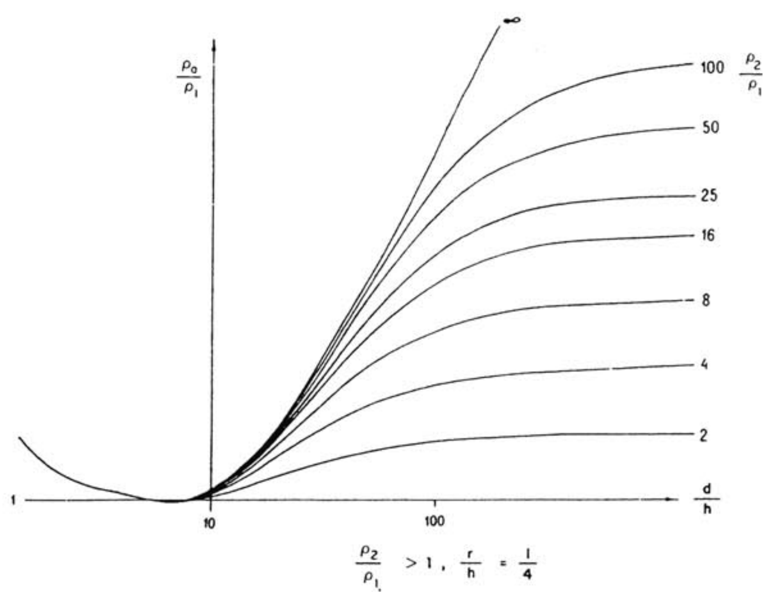


FIGURE 20. Two layer curves.

than the intercoil spacing. This contrasts very favourably with conventional resistivity techniques where it is necessary to expand an array to be several times the upper-layer thickness in order to fully resolve it.

(iii) The master curves shown in Fig. (19–20) were derived under the assumption of an infinitely fast turn-off of the transmitter current. In order to avoid performing a convolution the actual transmitter waveform should approximate this condition.

(iv) With conventional resistivity techniques one expands the array size from a value which is substantially less than the upper-layer thickness to a value much greater than the thickness in order to resolve it. In electromagnetic frequency-domain sounding it is necessary to analogously adjust the frequency from a value such that the skin-depth (defined in equation (3)) also varies from much less than to much greater than the upper-layer thickness. Since the skin-depth is inversely proportional to the square-root of the frequency this means that a large frequency range is necessary to adequately resolve the layer. The analogy for transient electromagnetic techniques is evident from Figs. (19–20) when it is realized that d is proportional to the square-root of time. In order to see the entire behaviour of the two-layered transient, measurement must be made over two decades of time and the time range must have been selected to correspond to the upper-layer diffusion thickness ($d/h \simeq 30$) at mid-range. Obviously this is not always possible and one often ends up working with a limited portion of the appropriate curve. Many useful techniques are given in the Russian literature for this situation and it is for this reason that it is of great importance to make accurate measurements over the limited range of the transient decay which in turn requires (a) the use of many narrow gates so as to ensure good time-resolution and avoid distortion, (b) that such gates cover a wide range of time and (c) to employ transmitters with large dipole moment so as to keep the signal-to-noise ratio as high as possible in view of the desirable narrow gate-widths.

(v) Finally, calculation of the late-stage behaviour when the two layers consist of a conductor over an insulator shows that the amplitude of the transient decay is proportional to the cube of the conductivity-thickness product of the upper layer, permitting accurate resolution of small changes in the thickness. This feature is of considerable significance in mapping the depth to an insulating basement through a sedimentary column. It is also significant that, for the case of either a uniform half-space or a horizontally stratified earth, current flow in transient sounding is always horizontal (in distinction to conventional resistivity techniques) so that anisotropy in the resistivity of horizontally layered sedimentary rocks is not a significant problem.

IV Thin Sheets

As is usually the case a thin sheet is defined as being so thin that there is no variation in current flow across the thickness of the sheet. In the case of sinusoidal excitation this means that the sheet thickness must be a small fraction of the skin depth in the sheet material; analogously for transient techniques sufficient time must have elapsed so that the current density is uniform throughout the cross-section of the sheet. With this constraint satisfied let us examine the behaviour of a vertical dipole transmitter located on the surface of such an (infinite) horizontal thin sheet. It can be shown that the current flow is given as a function of distance r from the transmitter and time by

$$J = \frac{3M}{\pi}\frac{1}{r^3}\frac{m}{(1+4m^2)^{5/2}} \tag{17}$$

where J = current density (amps/m)

$$m = \frac{t}{\mu S r}$$

S = sheet conductance (mhos)

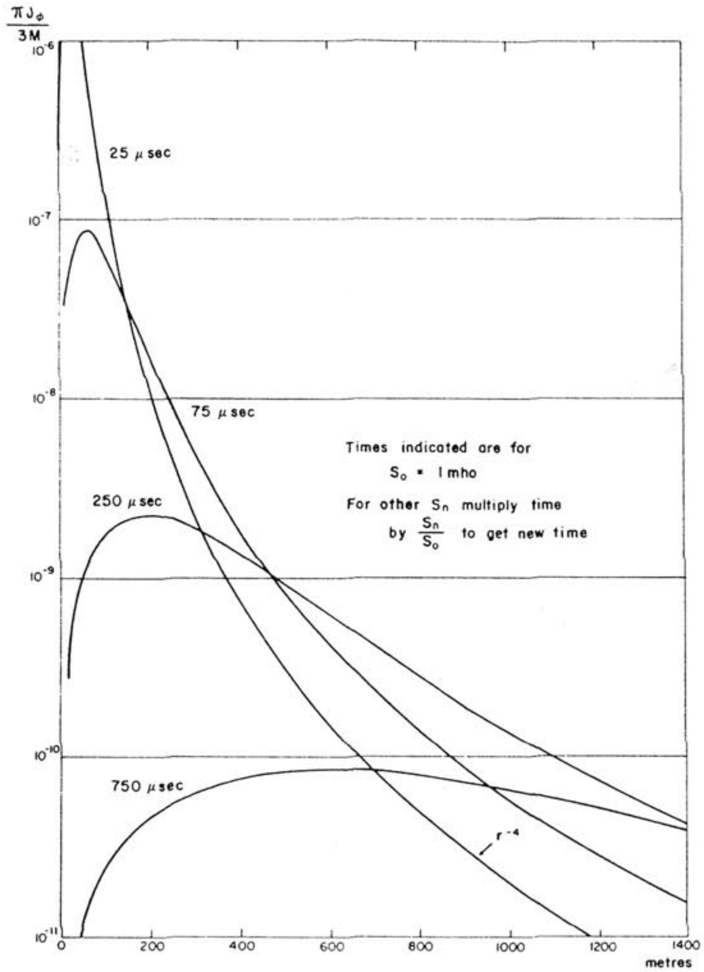


FIGURE 21. Current density – horizontal thin sheet.

Behaviour of this function, illustrated in Fig. 21, is similar to that of the homogeneous half-space; for any time t the current initially increases with distance from the transmitter, reaches a maximum and then decreases as r^{-4} (the dependence required to maintain the magnetic field at the value before current shut-off). Once again the initially induced current density is not a function of S, the conductivity-thickness product. Sheets with different values of S exhibit the same behaviour as long as the time scale is modified as indicated in Fig. 21.

By setting the derivative of equation (17) equal to zero it is a simple matter to solve for the location of maximum current-density as a function of time. The result is

$$t_{max} = \mu S r_{max} \tag{18}$$

Conversely there is an effective velocity defined as

$$v = \frac{r_{max}}{t} = \frac{1}{\mu S} \tag{19}$$

The position of maximum current moves out linearly with time (in distinction with the homogeneous half-space where it moved out as the square-root of time) and the effective velocity is determined by the conductivity-thickness product S. Calculation of the velocity for typical values of orebody conductances shows that once again it is quite low – for example at 100 mhos the velocity is 8 meters per msec.

A further difference between the homogeneous half-space and the thin sheet is that for the former immediately after turn-off the

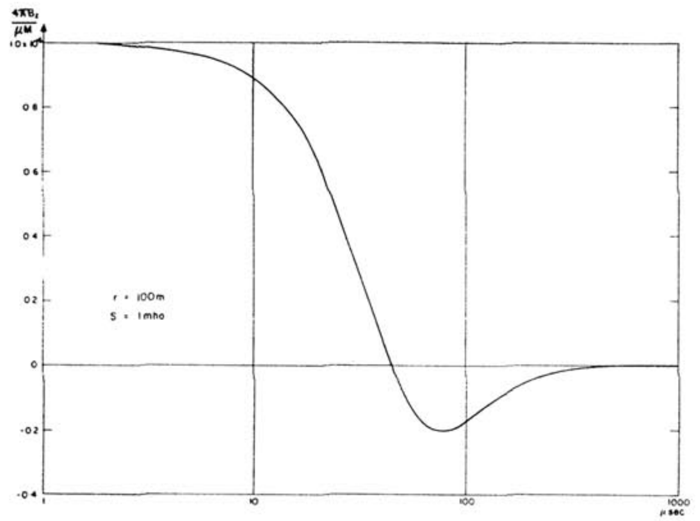


FIGURE 22. Vertical magnetic field component at surface of horizontal thin sheet.

surface currents at any point r fell off as r^{-4}; the current stayed at this value until the current ring passed underneath, after which time the current decreased monotonically to zero. In the case of the thin sheet at any location r the current initially increases, reaches a maximum value at the time given by equation (18), and then falls monotonically to zero.

The magnetic field components arising from the diffusing current ring also have the same general characteristics as those for the homogeneous half-space. For example Fig. 22 shows the vertical component of the magnetic field as a function of time. At short values of time the magnetic field is maintained at the value before turn-off. Since the primary field has been terminated this magnetic field arises solely from the currents flowing in the sheet. With the passage of time the deficiency of current between the transmitter and the observation point causes the vertical component to decrease, become zero and change polarity as the effective current moves beneath and passes beyond the observation point.

Again defining the late-time as that taken by the current to diffuse to a radial distance much greater than that of the observation point it can be shown that the vertical magnetic field and its time derivative are given at late-times by

$$B_z \simeq \frac{MS^3\mu^4}{16\pi t^3} \tag{20}$$

$$\text{and } \dot{B}_z \simeq \frac{-3MS^3\mu^4}{16\pi t^4} \tag{21}$$

Again the current wave diffuses without limit and the response is not exponential. Since for the thin sheet the current diffuses outwards at a faster rate (relatively) then for the homogeneous half-space the

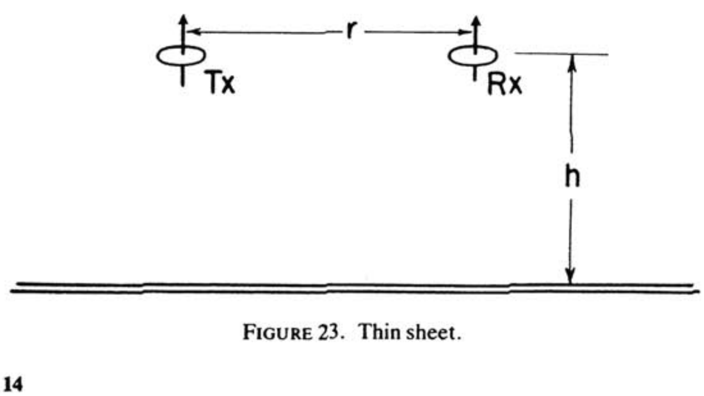


FIGURE 23. Thin sheet.

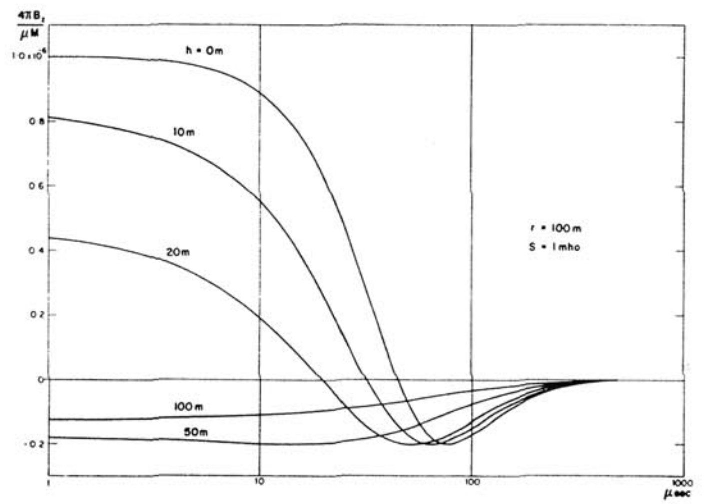


FIGURE 24. Vertical magnetic field component above horizontal thin sheet.

secondary magnetic field (and its time-derivative) decrease much more rapidly with time.

Measurement of the time-derivative of the vertical magnetic field at late-time yields S directly and it should be noted that the transient response of a thin sheet is singularly sensitive to the conductivity-thickness product being proportional to S^3.

Finally consider the situation where the vertical dipole is elevated to a height h above a (horizontal) thin sheet as shown in Fig. 23. In this case it can be shown that

$$B_z = \frac{\mu M}{4\pi r^3}\frac{1 - 8(\alpha t + H)^2}{[1 + 4(\alpha t + H)^2]^{5/2}} \tag{22}$$

$$\text{and } \dot{B}_z = \frac{-\mu M}{4\pi r^3}\frac{4\alpha(\alpha t + H)[9 - 24(\alpha t + H)^2]}{[1 + 4(\alpha t + H)^2]^{7/2}} \tag{23}$$

$$\text{where } \alpha = \frac{1}{\mu S r},\ H = \frac{h}{r} \tag{24}$$

The behaviour of the vertical component of the magnetic field is shown in Fig. 24. The affect of raising the transmitter and receiver dipoles above the sheet (i) reduces the initial amplitude of the vertical magnetic field and (ii) as seen from equation (22) linearly shifts the time scale by an amount $t_h = \mu Sh$; an event (such as the zero crossing) which happened at time t_o when both dipoles were located on the surface of the sheet now happens at time $t = t_o - t_h$.

The same translation occurs for the time derivative of the vertical magnetic field, which is illustrated in Fig. 25.

One would expect that, for dipoles raised above the sheet, at late-times the response will be the same as if the dipoles were on the sheet if by definition the current ring at such time is radially much further than the receiver distance and receiver/transmitter height. From equations (22) and (23)

$$\text{if } \frac{t}{\mu S} \gg h,\ \frac{t}{\mu S} \gg r \tag{25}$$

$$B_z \simeq \frac{M}{16\pi}\frac{\mu^4 S^3}{t^3} \tag{26}$$

$$\text{and } \dot{B}_z \simeq \frac{-3M}{16\pi}\frac{\mu^4 S^3}{t^4} \tag{27}$$

Neither quantity is a function of h and the expressions are indeed the same as for both dipoles located on the thin sheet.

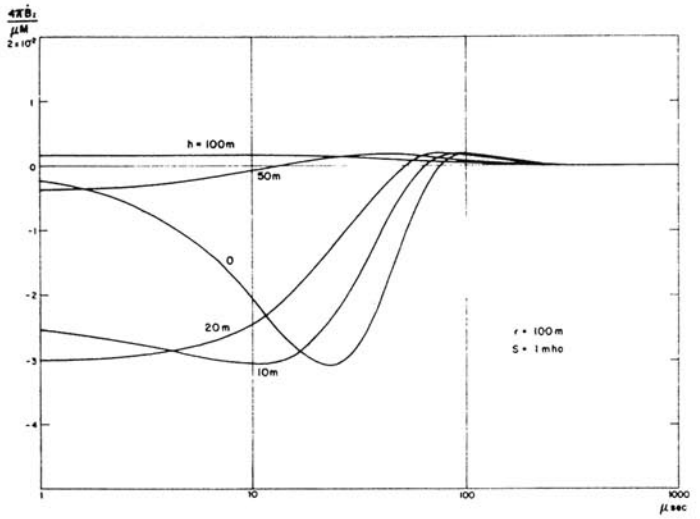


FIGURE 25. Derivative of vertical magnetic field above horizontal thin sheet.

Given S from such a measurement of $\dot{B}_z$ and knowing both r and t_0 (the time of the zero crossing of $\dot{B}_z$) it can be shown from equation (23) that h, the depth to the sheet, is given by

$$h = 0.6124r - \frac{t_0}{\mu S} \tag{28}$$

In order for the crossover to occur it is obvious that r must be chosen so that the right-hand side of this equation is positive and in general r is chosen to be as large as possible. This ensures, for given h and S, that the crossover occurs at realistically large values of time, particularly necessary for low values of S which cause the current ring to move out at great radial velocity.

V Further Comments

This concludes our discussion of typical models and their responses. It is seen that much information can be derived from transient measurements.

It has perhaps been noted that no mention was made of the semi-infinite vertical thin sheet so popular with horizontal loop systems. Because of the large transmitter loop size used with large transient systems it is less likely that targets whose dimensions are infinite compared with those of the transmitter loop will occur and a more valuable target model is the very oblate spheroid discussed earlier.

SOME MERITS AND DEFICIENCIES OF TRANSIENT ELECTROMAGNETIC TECHNIQUES

One sometimes hears that transient measurements are essentially quadrature-phase determinations since measurement is made in the absence of the primary field. Although it is true that targets which exhibit in-phase response only (such as non-conductive bodies with $\mu > \mu_0$) also yield no response after termination of the primary field it will be shown below that in general the transient method is responsive to both the in-phase and quadrature-phase components of the target response.

Furthermore one hears that since transient and multi-spectral frequency-domain responses are related through the Fourier transform such techniques are equivalent. Whilst theoretically correct this statement ignores the important influence of the major source of noise for each technique, the net result of which is that very often they are not in a practical sense equivalent and transient techniques can offer a major advantage.

Consider the transmitter current waveform in Fig. 26a. This waveform is easily shown to be the Fourier sum of a fundamental and an infinite series of *odd* harmonics

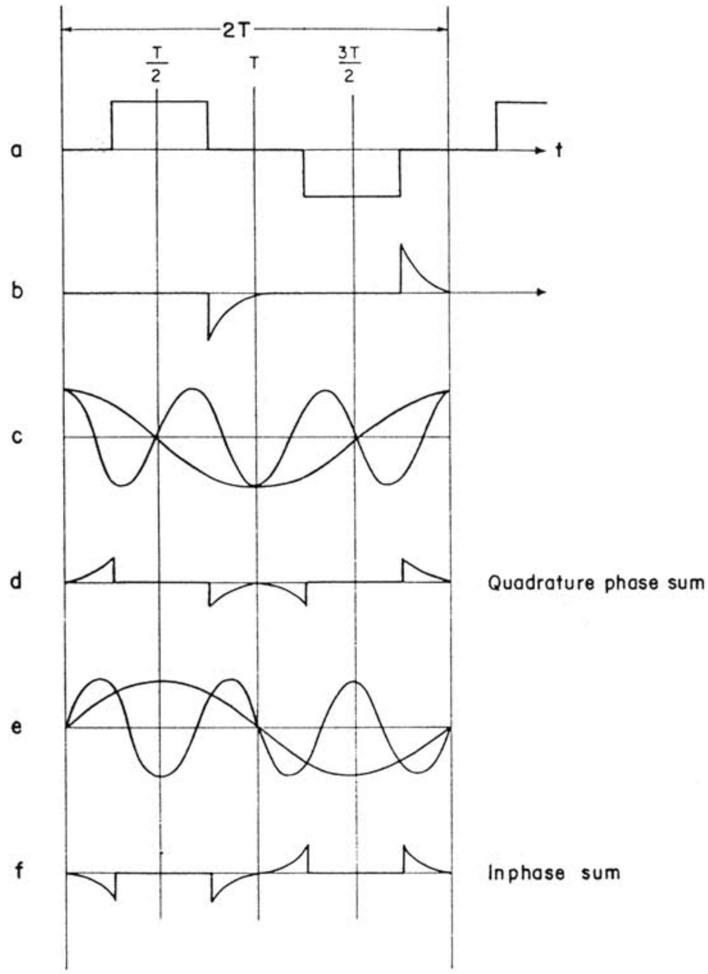


FIGURE 26. Synthesis of transient response.

$$f(t) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{2}\right) \frac{\sin\left(\frac{n\pi}{4}\right)}{\frac{n\pi}{4}} \sin n\omega t \tag{29}$$

Each of these frequencies excites the target which we assume to have an exponential decay with time-constant which is short compared with one-quarter of the period of the fundamental (thus ensuring both in-phase and quadrature-phase response). The receiver gates out the transients which occur during the transmitter on-time; those which remain for processing are shown in Fig. 26b. Since these transients arise from the transmitter excitation it must also be possible to synthesize this waveform from an infinite series of odd harmonics. The question arises as to whether there will be only cosine terms (quadrature-phase) or both cosine and sine (in-phase) terms in this series.

Consider a series of cosines at fundamental and odd harmonic frequencies, an example of the first two terms of which is shown in Fig. 26c. Examination of this figure shows that, as long as only the odd harmonics are present, the infinite sum of such harmonics (at arbitrary amplitudes) must exhibit even symmetry about T. Consider further the interval from 0 to T. Within this interval odd symmetry must be exhibited about T/2. Likewise in the interval T to 2T odd symmetry must be exhibited about 3T/2. The synthesized waveform shown in Fig. 26d satisfies these requirements.

Consider next a series of odd harmonic sine terms, the first two terms of which are shown in Fig. 26e. Their infinite sum must exhibit odd symmetry about T and, in the smaller intervals, even symmetry about T/2 and 3T/2. Such a synthesized waveform is shown in Fig. 26f. Now in the interval 0 to T/4 the response shown in Fig. 26b is zero. Therefore within this interval the waveforms shown in Fig. 26d and Fig. 26f must be everywhere equal, since their sum is everywhere zero. Furthermore the waveforms shown in Fig. 26d and Fig. 26f between 3T/4 and T/4, 7T/8 and 2T must be everywhere the same. If the in-phase response were everywhere zero the quadrature-phase would have to be zero as well. Evidently the quadrature and in-phase components are equally important in synthesizing the final waveform.

Transient systems are definitely not quadrature-phase systems; they generally respond about equally to the in-phase and quadrature-phase components from the target.

In fact the principal advantage of the transient electromagnetic system is that it allows measurement of many of the in-phase characteristics of the response of a target in the absence of the primary field. To understand the significance of in-phase measurement we refer to a recent article by Kaufman (7) in which he asserts that the most diagnostic exploration system is the one which is the most sensitive to small changes in conductivity, ie. has a response which is proportional to conductivity raised to a high power. He reviews the fact that conventional (DC) resistivity surveys respond functionally to conductivity contrasts as

$$\frac{\left(1 - \frac{\sigma_t}{\sigma_b}\right)}{\left(1 + \frac{2\sigma_t}{\sigma_b}\right)} \tag{30}$$

where σ_t = target conductivity
σ_b = background conductivity

which function is insensitive to changes in conductivity at medium or high conductivity contrasts. Such systems are therefore inherently not diagnostic.

Turning to frequency-domain methods he shows that at low frequencies where both the in-phase and quadrature-phase response are increasing with target conductivity the quadrature-phase component responds linearly to conductivity variations. This is an improvement over conventional resistivity but better can be achieved with the in-phase component, for in this case at low frequencies the system response is of the order σ^2. In-phase measurements are truly more diagnostic but this advantage is seldom fully realized since the major source of noise in such measurements results from inadvertent changes in the inter-coil spacing. Since the in-phase component arising directly from the transmitter is proportional to r^{-3} (where r is the intercoil spacing) small changes in this spacing have a large effect on the signal-to-noise ratio. Such noise is completely absent in the case of transient systems in which measurement is made in the absence of the primary field. Yet they are still, as we have seen above, sensitive to the in-phase components. We should therefore expect such systems to be able to take full advantage of the diagnostic value of the in-phase components and this is indeed the case.

As our first example consider the infinite horizontal thin sheet referred to earlier. At low frequencies it can be shown that the quadrature-phase component is proportional to S. Conversely the in-phase component is proportional to S^2 and as we have seen, the transient response is proportional to S^3. Obviously in the context of the last paragraph the transient response is the most diagnostic. A less-than-obvious advantage of transient systems lies in the way in which the in-phase and quadrature-phase components of the target response are effectively synthesized to make the final transient waveform.

Kaufman continues to explore the relative merits of various electromagnetic techniques for the exploration picture shown in Fig. 27. A large, shallow, moderately conductive bounded overburden overlies a deep, small, but intrinsically more conductive target. As a result of its proximity to both the transmitter and receiver the overburden yields a large response to quadrature-phase systems, even though not particularly conductive.

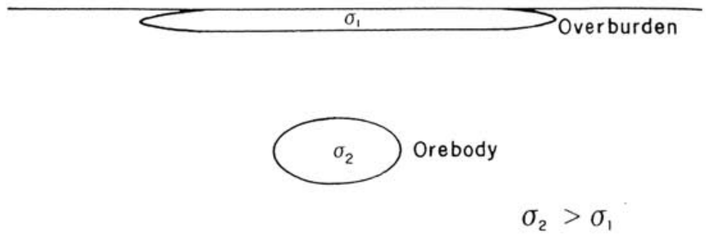


FIGURE 27. Orebody beneath conductive overburden.

The situation is substantially improved for the case of in-phase measurements since the influence of conductivity (relative to proximity) is enhanced. However such improvement requires that instrumental noise be of low order (optimally less than the response from the overburden).

The ultimate performance is achieved by the use of transient techniques. The early-time response from the overburden can be very large but if the decay rate is faster than that of the orebody one simply has to wait a sufficient length of time for the masking effects of the overburden to disappear and the orebody to appear. The only requirement is that the transient decay of the orebody be slower than that of the overburden so that the latter eventually disappears. Unfortunately by the time that this happens the response from the orebody may be so low as to be undetectable against the system noise. But the noise in this case is external (man-made interference, spherics) and there is always one way to overcome it – brute force, ie. a large high-powered transmitter.

In the case of in-phase systems increases in transmitter power have no effect – both the signals and the noise linearly increase with transmitter dipole moment. For transient systems the signal-to-noise ratio is almost invariably linearly related to transmitter power. The higher the power (ie. the higher the primary inducing magnetic field) the longer one can afford to wait for the various effects (overburden, etc.) to sort themselves out. In the case of bounded conductors with adequate transmitter power everything comes to he who waits!

A disadvantage of transient systems is that by virtue of being broadband they are sensitive to noise and interference of the type referred to above. For this reason a large transmitter dipole moment is desirable and since resonant techniques are not available with pulses the transmitters tend to be heavy and the transmitter loops of large area. For this reason it is not currently possible to make a system that is truly portable in the sense of a conventional horizontal loop system.

In the example just given it was assumed that the "geological noise" also consisted of a confined conductor so that the response was exponential with time. Consider now the situation where the target (in this example, a sphere) is imbedded below the transmitter in a conductive host-rock at a depth z (Fig. 28). The same loop is also used as the receiver coil. At early times, before the diffusing current arrives at the location of the target the secondary fields will be entirely those of the homogeneous half-space as described in the previous section. As the diffusing current approaches the target the changing magnetic field of the current ring generates the eddy-current flow in the target (the transmitter has long since been shut off). During this time there is a good deal of electromagnetic coupling between the ring and the target. As time progresses the current ring diffuses onwards and the currents in the (presumed relatively conductive) target commence to decay and later to diffuse outwards into the conductive half-space. At still later times the currents from the target will have dissipated to zero and once again the measured secondary fields will be essentially those of the homogeneous half-space (an exponential decay is much faster than $t^{-3/2}$).

It can be shown (8) that, for targets of such a size, location, and

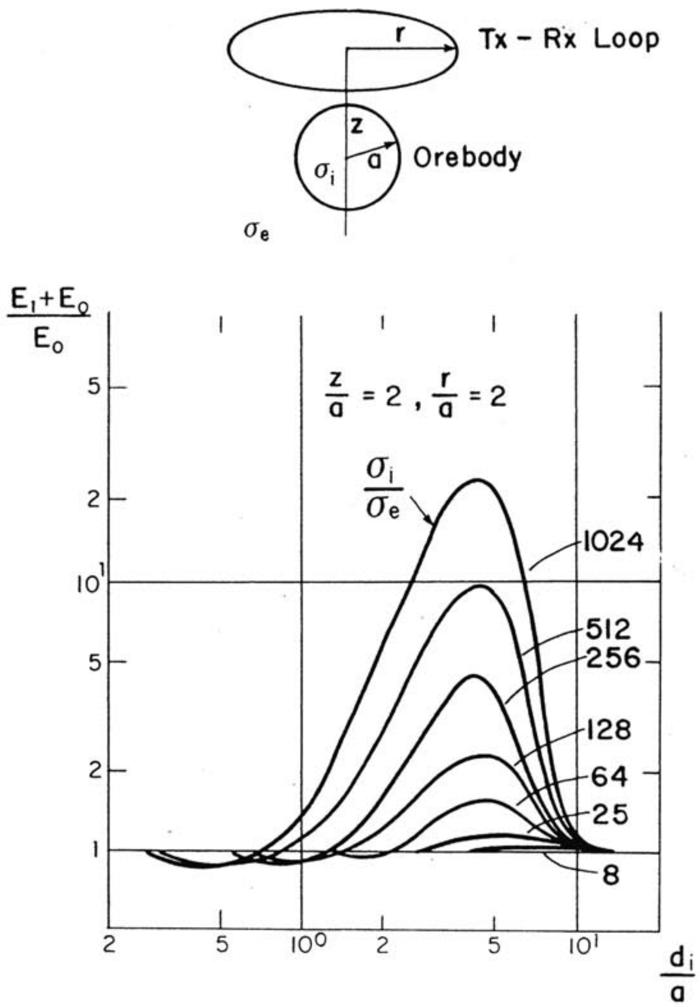


FIGURE 28. Combined response from orebody and conductive host rock (8).

geometry as to give a reasonably large response compared to the homogeneous half-space, there is an interval of time during which (i) the response of the orebody and the half-space are relatively uncoupled – ie. the total response is essentially the sum of the two individual responses, and (ii) the response from the target is exponential with the free space time-constant so that measurements made during this time are diagnostic of the target. An example of such a response is shown in Fig. 28. The quantity plotted as a function of d_i/a is the ratio of the total emf induced in the loop when both target and half-space are present to the emf when just the half-space is present. The figure clearly indicates the "window" during which measurements are quite sensitive to the target parameters, as well as the early and late-time behaviour referred to above.

Similar calculations carried out for conventional multi-spectral frequency-domain systems show that the quadrature-phase response does not exhibit the type of behaviour shown in Fig. 28 (now as a function of frequency) whereas the in-phase response does exhibit analogous characteristics to the transient response. Once again the advantage of the transient technique is that it permits virtually noise-free (assuming no limit to the transmitter dipole moment) measurements of the essential features of the in-phase response.

An interesting effect can occur if the current ring actually impinges on the target. Consider again Fig. 1 which shows a sheet conductor now assumed to be embedded in a homogeneous half space of conductivity σ_e. At early times such that d_e/r is small (r is now the distance to the target) the magnetic field at the target remains exactly as it was before shut-off. The only current flow that occurs in the vicinity of the target is the surface current that flows on the surface above it. With the passage of time the current ring diffuses outward with the result that the changing magnetic field from the moving current causes induced eddy currents within the target. Two situations can occur; (i) the target is located sufficiently far from the path of the diffusing current ring at all times so that the only significant effect in the target is the induced eddy currents arising from the changing magnetic field or (ii) the target may actually be in the path of the diffusing current in which case the current will be substantially distorted as it envelopes the target. This is an example of what has recently been called "current gathering". It is obviously a question of degree since there is always some conductive interaction between the diffusing current and the target. Measurements made of the magnetic field or its time derivative on the surface above the target will be additionally affected by current gathering.

Finally having accepted the fact that the transmitter loop of a transient system, particularly a high-powered one, is rather large there is another major advantage associated with such systems which is the high degree of flexibility that is available in exploring the geometrical aspects of the target response. With complete freedom one moves the receiver about, making three-axis measurements of the time derivative of the *target* response only, rather than of a complicated vector function of both the transmitter (primary) magnetic field and the target response.

An obvious extension of this freedom applies to borehole geophysics. In conventional frequency-domain methods lowering the sonde down a borehole accurately depicts the fall-off with distance of the primary magnetic field with, occasionally, a small anomaly superimposed on the large primary signal. With transient techniques there is nothing – unless an anomaly.

SUMMARY

Transient electromagnetic techniques offer significant advantages over multi-spectral frequency-domain measurements. The latter have been well treated in the literature; relatively little information is available about transient responses. It is hoped that this technical note will help to rectify the unbalance.

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